Calculus Warm Up #5-5

Use the first and/or second derivative to find any extrema for the graph of

$$f(x) = x^{2/3} - 3$$

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

undef for $x = 0$

f': -1, 0, 1

$-$ $\quad$ $+$

$\searrow$ $\quad$ $\nearrow$

Min @ $(0, -3)$

Groups:

Go over #19 and 23.

1) Compare first and second derivatives. Fix any disagreements.

2) Compare critical #'s and intervals in your table. Fix any disagreements.

3) Compare the conclusions and fix any disagreements.

4) Compare graphs for details.

26) $f(x) = x^4 - 4x^3$

Places of interest and intervals between	test #'s on the interval	$f'(x)$ +, - or 0	$f''(x)$ +, - or 0	Conclusion
$(-\infty, 0)$	-1	−	+	f decr., conc up
$x = 0$		0	0	PI @ (0,0)
$(0, 2)$	1	−	−	f decr., conc dwn
$x = 2$			0	PI @ (2,-16)
$(2, 3)$	2.5	−	+	f decr., conc up
$x = 3$		0	+	Min @ (3,-27)
$(3, \infty)$	4	+	+	f incr., conc up

4.5

-Limits at infinity

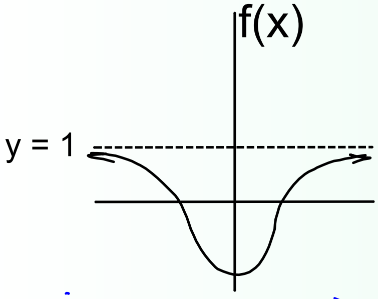
-Horizontal asymptotes

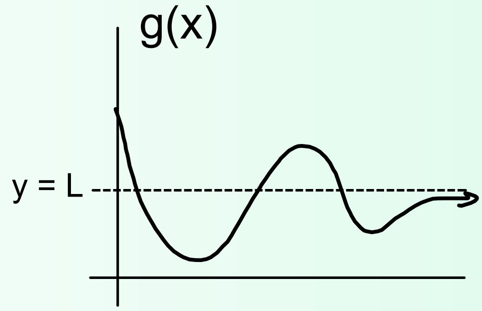
Remember Limit Properties:

$$\lim_{x \to \infty} [f(x) + g(x)] = \lim_{x \to \infty} f(x) + \lim_{x \to \infty} g(x)$$

$$\lim_{x \to \infty} [f(x)\, g(x)] = \left(\lim_{x \to \infty} f(x)\right)\left(\lim_{x \to \infty} g(x)\right)$$

Horizontal Asymptotes:





$\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to \infty} f(x) = 1$ $\lim_{x \to \infty} f(x) = L$

Both fit the Definition of a Horizontal Asymptote (p. 183)

Characteristics to consider and compare to vertical asymptotes:

1) Limit exists, it = L, a real number
2) Function can be defined on y = L.
3) There can be at most 2 horizontal asymptotes.

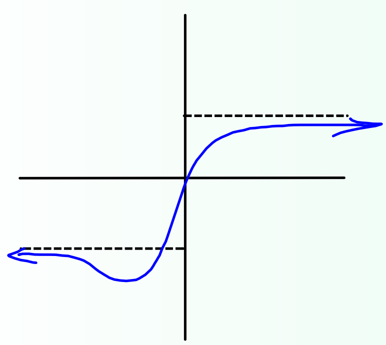
DEFINITION OF HORIZONTAL ASYMPTOTE

If

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L$$

then the line $y = L$ is called a **horizontal asymptote** of the graph of f.

Example of 2 Horizontal Asymptotes:



Limits at Infinity:

$$\lim_{x\to\infty} \frac{c}{x^r} = 0$$

c = real # constant
r = positive, rational #

Examples:

1. $\lim_{x\to\infty} \frac{3}{x^2}$

$= \frac{3}{\infty}$

$= 0$

2. $\lim_{x\to\infty} \frac{-5}{\sqrt{x}}$

$-\frac{5}{\infty}$

0

Evaluate:

$$\lim_{x\to\infty} \frac{2x-1}{x+1}$$

Needs fussing. $\longrightarrow$ $\cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \frac{2(\infty)-1}{\infty+1}$

$= \frac{\infty}{\infty}$:(

limit can not be determined.

$\lim_{x\to\infty} \frac{\cancel{\frac{2x}{x}} - \frac{1}{x}}{\cancel{\frac{x}{x}} + \frac{1}{x}}$

$= \lim_{x\to\infty} \frac{2-\frac{1}{x}}{1+\frac{1}{x}}$

$\frac{2-0}{1+0}$

$\boxed{2}$

Find the following limits.

(a) $\lim_{x\to\infty} \frac{2x+5}{3x^2+1}$ (b) $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$ (c) $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

$= \lim_{x\to\infty} \frac{\frac{2x}{x^2} + \frac{5}{x^2}}{\frac{3x^2}{x^2} + \frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{2}{x} + \frac{5}{x^2}}{3 + \frac{1}{x^2}}$

$= \frac{0+0}{3+0}$

$= \boxed{0}$

$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2} + \frac{5}{x^2}}{\frac{3x^2}{x^2} + \frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{2 + \frac{5}{x^2}}{3 + \frac{1}{x^2}}$

$\frac{2+0}{3+0}$

$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3} + \frac{5}{x^3}}{\frac{3x^2}{x^3} + \frac{1}{x^3}}$

$\lim_{x\to\infty} \frac{2 + \frac{5}{x^3}}{\frac{3}{x} + \frac{1}{x^3}}$

$\frac{2+0}{0+0}$

$\frac{2}{0}$!!

Determine the following limits.

(a) $\lim_{x\to\infty} \frac{3x-2}{\sqrt{2x^2+1}}$ (b) $\lim_{x\to-\infty} \frac{3x-2}{\sqrt{2x^2+1}}$

$\sqrt{x^2} = -x$

$\lim_{x\to-\infty} \frac{\frac{3x}{-x} - \frac{2}{-x}}{\sqrt{\frac{2x^2}{x^2} + \frac{1}{x^2}}}$

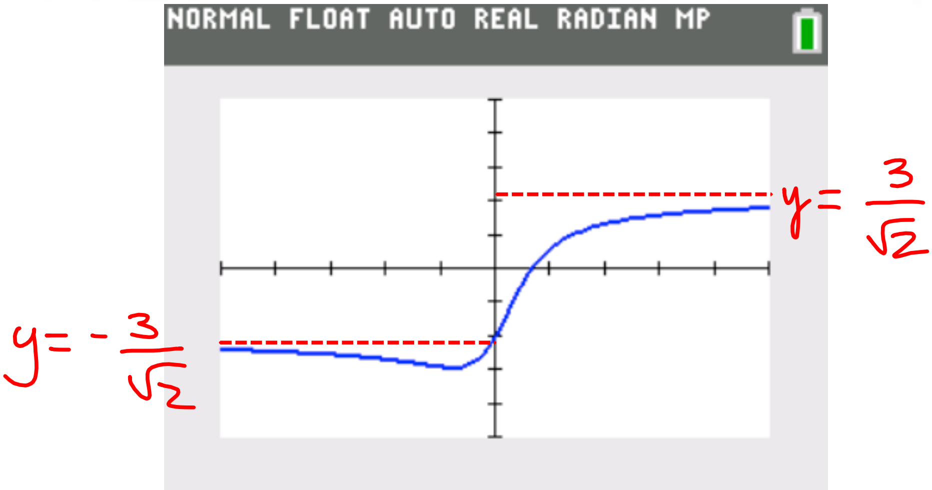
$\lim_{x\to-\infty} \frac{-3 + \frac{2}{x}}{\sqrt{2 + \frac{1}{x^2}}}$

$\frac{-3+0}{\sqrt{2+0}}$

$-\frac{3}{\sqrt{2}}$

Determine the following limits.

(a) $\lim_{x \to \infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}}$ (b) $\lim_{x \to -\infty} \dfrac{3x - 2}{\sqrt{2x^2 + 1}}$



EXAMPLE 5 An application involving oxygen levels

Suppose that $f(t)$ measures the level of oxygen in a pond, where $f(t) = 1$ is the normal (unpolluted) level and the time t is measured in weeks. When $t = 0$, organic waste is dumped into the pond, and as the waste material oxidizes, the amount of oxygen in the pond is given by

$$f(t) = \frac{t^2 - t + 1}{t^2 + 1}.$$

What percentage of the normal level of oxygen exists in the pond after 1 week? After 2 weeks? After 10 weeks? What is the limit as t approaches infinity?

HW:

p. 188 # 9 - 45 eoo

Quiz: 4.1 - 4.4   Monday
Extrema on open and closed intervals
Mean Value Theorem
Increasing/decreasing intervals
Concavity

Extra Credit on the quiz will be about position and velocity.